Intervisibility

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What is the height (a) of a point C which can just cut off the view of point B (height h_2) from point A (height h_1)?

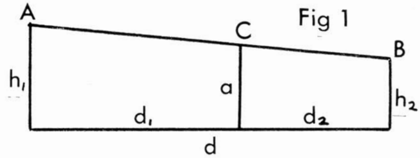

Fig 1

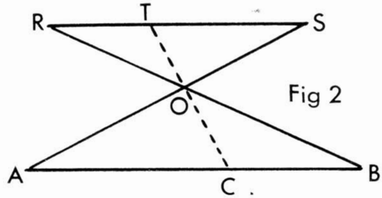

Fig 2

Over short distances where the curvature of the earth can be neglected (say up to 5 to 10 miles) intervisibility is a simple linear geometrical problem (see Fig. 1), where:

$$a = \frac{1}{d}(h_1 d_2 + h_2 d_1)$$

Figure 2 shows a construction which can be carried out on any contour map. Join point A (height h_1) to point B (height h_2). Parallel to AB draw a line RS of length proportional to the height difference $h_1 - h_2$ and construct a scale of heights along RS, with height h_2 at R and h_1 at S. Join R to B and S to A; the lines intersect at O. To check what happens at point C (height a), join CO and produce to cut MN at T. If the scale value at T is less than a then B cannot be seen from A, if the value at P exceeds a then A and B are intervisible.

For greater separations the curvature of the earth has to be taken into account. The following expression can be derived from Fig. 3

$$a = \frac{1}{d}(h_1 d_2 + h_2 d_1) - \frac{d_1 d_2}{2r}$$

(Certain assumptions have been made to obtain this simple expression, but these are justified taking into account the magnitudes of the various quantities involved.)

From the above we can derive the well-known result for the distance of the visible horizon. Putting $a = 0$, $h_1 = h_2 = H$ and $d_1 = d_2 = D$ we find:

$$H = \frac{D^2}{2r}$$

and if figures are put in for r

$$H = 0{\cdot}66\ D^2 \qquad (H \text{ in feet, } D \text{ in miles})$$

or

$$H = 0{\cdot}0785\ D^2 \qquad (H \text{ in metres, } D \text{ in kilometres})$$

Up to now it has been assumed that the line of sight from A to B is a straight line. However, because of refraction due to temperature gradients in the earth's atmosphere, this is never the case; the line of sight is in fact curved as shown dotted in Fig 3. AB is the theoretical straight line joining the two points; the curved line shows the actual line of sight due to refraction, so that an observer at A has actually to look in the direction AD in order to see B.

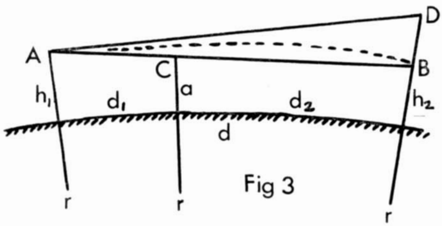

Fig 3

To take account of this additional complexity the formulae have to be modified by a factor $(1-2K)$ where K is known as the coefficient of refraction—found to have a mean value of 0·07.

The intervisibility formula becomes:

$$a = \frac{1}{d}(h_1 d_2 + h_2 d_1) - \frac{(1 - 2K)d_1 d_2}{2r}$$

$$a = \frac{1}{d}(h_1 d_2 + h_2 d_1) - 0{\cdot}57\ d_1 d_2 \qquad (h \text{ in feet, } d \text{ in miles})$$

or

$$a = \frac{1}{d}(h_1 d_2 + h_2 d_1) - 0{\cdot}0675\ d_1 d_2 \qquad (h \text{ in metres, } d \text{ in kilometres})$$

The distance of the visible horizon is given by:

$$H = (1 - 2K)\frac{D^2}{2R}$$

$$H = 0{\cdot}57\ D^2 \qquad (H \text{ in feet, } D \text{ in miles})$$

or

$$H = 0{\cdot}0675\ D^2 \qquad (H \text{ in metres, } D \text{ in kilometres})$$

The value of the refractive index and thus the coefficient of refraction varies widely in unusual conditions of temperature gradient. Abnormal refraction conditions account for many of the cases of unusual sightings which have been reported from time to time.

The effects of refraction are of particular importance in the measurement of the heights of mountains where it is usually impossible to determine the characteristics of the whole length of the optical path. Observations of vertical angles are made at 'the time of minimum refraction', near midday, when the day-to-day variation of temperature gradient is found to be the least. A value for the temperature gradient has to be assumed. Even so, the estimate of refraction, says Gulatee, can be in error by as much as 25 per cent. The observations to Mount Everest from the plans of India necessitated a refraction correction of 1375 ft.

References

Gulatee, B. L., 'Mount Everest—Its mass and height'. *H.J. 17* 131.
Jenkins, G. G., *Hill Views from Aberdeen*. Wyllie, Aberdeen, 1917.
Parker, J. A., *'Curvature and Visibility'. S.M.C.J. 20* 317.

66 *Westwards from Monte Disgrazia (foreground, Monte Disgrazia west north-west peak—middle distance, Bregaglia peaks—far distance, Adula and Oberland peaks.)*
Photo: B. R. Goodfellow